Factorization of holomorphic eta quotients

ABSTRACT:
Unlike integer factorization, a reducible holomorphic eta quotient may not factorize uniquely as a product of irreducible holomorphic eta quotients. We conjecture the occurrence of a certain type of factor of a holomorphic eta quotient whenever it is reducible and we give a short sketch of our proof of this conjecture for all prime power levels. This also implies that rescaling and Atkin-Lehner involutions of irreducible holomorphic eta quotients of prime power levels are irreducible. In particular, our conjecture implies an irreducibility-checking algorithm for holomorphic eta quotients. We show that even without assuming this conjecture, irreducibility of an eta quotient could be checked in finite time. We also show that there are finitely many simple holomorphic eta quotients of a given level and provide a bound on the weights of such eta quotients of a given level. Finally, we construct an infinite family of irreducible holomorphic eta quotients of prime power levels.

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